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Robust H_∞ Filter Design for Uncertain Linear Systems Over Network with Network-Induced Delays and Output Quantization

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Abstract

This paper investigates a convex optimization approach to the problem of robust H_∞ filtering for uncertain linear systems connected over a common digital communication network. We consider the case where quantizers are static and the parameter uncertainties are norm bounded. Firstly, we propose a new model to investigate the effect of both the output quantization levels and the network conditions. Secondly, by introducing a descriptor technique, using Lyapunov-Krasovskii functional and a suitable change of variables, new required sufficient conditions are established in terms of delay-dependent linear matrix inequalities (LMIs) for the existence of the desired network-based quantized filters with simultaneous consideration of network induced delays and measurement quantization. The explicit expression of the filters is derived to satisfy both asymptotic stability and a prescribed level of disturbance attenuation for all admissible norm bounded uncertainties.

Keywords: Filter design, network, output quantization, delay

1 Introduction

Networked control systems (NCS) in which control and communication issues are combined together, and all the delays and limitations of the communication channels between sensors, actuators, and controllers are taken into account has become an enabling technology for many military, commercial and industrial applications. In practice, due to the finite switching speed of amplifiers or finite speed of information processing, time delays including delays in the state or in the derivative of the state are often encountered in hardware implementation, which may be a source of oscillation, divergence, and instability in system Gao and Chen (2007); Gao et al. (2007); Gao and Wang (2003); Karimi and Gao (2009a); Karimi and Gao (2009b); Karimi and Gao (2008); Karimi et al. (2008); Karimi and Maass (2009); Lam et al. (2005); Park (1999). The study of NCSs is an interdisciplinary research area, combining both network and control theory. That is, in order to guarantee the stability and performance of an NCS, analysis and design tools based on both network and control parameters are needed. Modeling, analysis, and design of NCSs have received increasing attention in recent years, see Ishii and Francis (2002); Zhivoglyadov and Middleton (2003). In an NCS, sensor and/or controller data are transmitted through network channels. NCSs can be applied to a wide variety of engineering systems including manufacturing plants, aircrafts, automobiles, etc. In this correspondence, an NCS consists of a plant, sensors, actuators, and a controller, as in a typical control system. However, in an NCS, the sensor data packets reach the controller, and controller data packets arrive at the actuators via network channels. In such a setting, the network load and the limited communication bandwidth can cause network-induced delays. Recently, the robust H_∞ control problem for a class of networked systems with ran-

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dom communication packet losses has been studied by Z. Wang and Liu (2007).

However, due to network bandwidth restriction, the insertion of communication network in the feedback control loop inevitably leads to communication delays and makes the analysis and design of NCSs complex. Communication delays can deteriorate the performance of NCSs and even can destabilize the systems when they are not considered in the design of NCSs. So far, a variety of efforts have been devoted to analyzing NCSs with communication delays (see, e.g., Branicky et al. (2000); Gao and Chen (2008); Gao et al. (2008); Hu and Zhu (2003); Kim et al. (2003); Matveev and Savkin (2001); Montestruque and Antsaklis (2003), Nilsson et al. (1998); Wong and Brockett (1999); Yu et al. (2003); Yue et al. (2004); Yue et al. (2005), Yue and Han (2006); Zhang et al. (2001) and the references therein). Specifically, Branicky et al. (2000) and Zhang et al. (2001) analyzed the stability of NCSs and obtained stability regions using a hybrid systems technique. Kim et al. (2003) presented linear matrix inequality (LMI) conditions for obtaining maximum allowable delay bounds, which guarantee the stability of NCSs. Based on Lyapunov-Razumikhin function method, Yu et al. (2003) presented conditions on the admissible bounds of data packet loss and delays for NCSs in terms of LMIs. Based on stochastic control theory, optimal controller design of NCSs with stochastic network delays was investigated in Nilsson et al. (1998); Matveev and Savkin (2001); Hu and Zhu (2003). For other control schemes, we refer readers to the survey Tipsuwan and Chow (2003). To reduce the network traffic load, Montestruque and Antsaklis (2003), Montestruque and Antsaklis (2004) proposed a model-based control scheme for NCSs without/with delays. Necessary and sufficient conditions for the exponential stability of discrete-time and continuous-time NCSs without/with communication delays were established in both cases of state feedback and output feedback. However, they did not present any method for controller design when communication delays are considered. Moreover, it is in general not an easy task to design the controller based on their conditions. Mu et al. (2004) proposed an improved model based control scheme for NCS without/with delays and presented conditions for exponential stability together with controller design procedures. Particularly, an impulsive model based control scheme for discrete-time NCSs without communication delays was discussed. Recently, the problem of output feedback control for networked control systems (NCSs) with limited communication capacity was studied by Tian et al. (2008).

In this paper, we contribute to the further development of a convex optimization approach to the problem

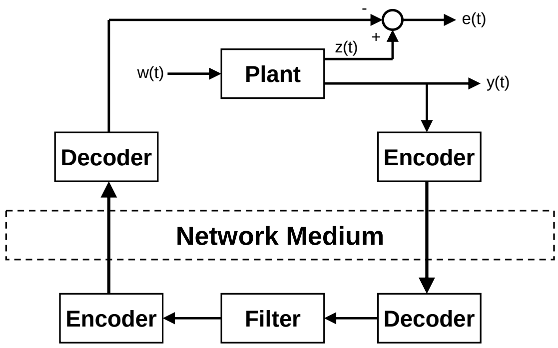

Figure 1: A typical network-based filter

of robust H_∞ filtering for uncertain linear systems connected over a common digital communication network. The network can be considered as depicted in Fig. 1. Here, we consider the case where quantizers are static and the parameter uncertainties are norm bounded. Firstly, we propose a new model to investigate the effect of both the output quantization levels and the network conditions. Secondly, by introducing a descriptor technique, using Lyapunov-Krasovskii functional and a suitable change of variables, new required sufficient conditions are established in terms of delay-dependent linear matrix inequalities (LMIs) for the existence of the desired network-based quantized filters with simultaneous consideration of network induced delays and measurement quantization. The explicit expression of the filters is derived to satisfy both asymptotic stability and a prescribed level of disturbance attenuation for all admissible norm bounded uncertainties.

The rest of this paper is organized as follows. Section 2 is the problem formulation and related preliminaries. In section 3, we investigate the robust H_∞ performance analysis of the filtering error system. The robust H_∞ filter design problem with normbounded uncertainties is addressed in Section 4 and the result is obtained based on the notion of asymptotic stability and LMIs. A numerical example is provided to illustrate the effectiveness of the approach presented in this paper in Section 4. And, we conclude the paper in Section 5.

The notations used throughout the paper are fairly standard. I_n and 0_n represent, respectively, n by n identity matrix and n by n zero matrix; the superscript T stands for matrix transposition; $\Re^n$ denotes the n-dimensional Euclidean space; $\Re^{n\times m}$ is the set of all real m by n matrices. The matrices $\hat{I}$ and $\tilde{I}$ are defined, respectively, as $\hat{I} := [I, 0]$ and $\tilde{I} := [0, I]$. $\|.\|$ refers to the Euclidean vector norm or the induced matrix 2-norm and $diag\{\cdots\}$ represents a block diagonal matrix. $\lambda_{min}(A)$ and $\lambda_{max}(A)$ denote, respectively, the smallest and largest eigenvalue of the square matrix A. The operator $sym\{A\}$ denotes $A + A^T$ and $[.]$ is the operation of round. The notation $P > 0$ means that P is real symmetric and positive definite and the symbol $*$ denotes the elements below the main diagonal of a symmetric block matrix. In addition, $L_2[0, \infty]$ is the space of square-integrable vector functions over $[0, \infty]$. Matrices, if the dimensions are not explicitly stated, are assumed to have compatible dimensions for algebraic operations.

2 System description

Consider the following continuous-time system with time-varying structured uncertainties:

$$\dot{x}(t) = (A + \Delta A(t))x(t) + (B + \Delta B(t))w(t), \quad (1)$$
$$y(t) = Cx(t), \quad (2)$$
$$z(t) = Gx(t) \quad (3)$$

where $x(t) \in \Re^n$ is the state vector, $y(t) \in \Re^m$ is the measured output, considered as the control input; $w(t) \in \Re^l$ and $z(t) \in \Re^r$ are the disturbance and the signal to be estimated, respectively. The coefficient matrices A, B, C, G are real matrices with appropriate dimensions. The time-varying structured uncertainties $\Delta A(t)$ and $\Delta B(t)$ are said to be admissible if the following form holds

$$\begin{bmatrix}\Delta A(t) & \Delta B(t)\end{bmatrix} = M_1 F(t) \begin{bmatrix}L_a & L_b\end{bmatrix} \quad (4)$$

where L_a, L_b are constant matrices with appropriate dimensions; and $F(t)$ is an unknown, real, and possibly time-varying matrix with Lebesgue measurable elements, and its Euclidean norm satisfies

$$\|F(t)\| \leq 1, \ \forall t \quad (5)$$

We are interested in investigating the stability property of systems when the observer undergoes quantization and delays. This kind of problem arises in scenarios in which a finite bandwidth channel lies in the feedback loop and introduces a delay.

In this paper, a quantizer means a piecewise constant function $q : \Re^p \to Q$, where Q is a finite subset of $\Re^l$. This leads to a partition of $\Re^l$ into a finite number of quantization regions of the form $\{z \in \Re^l : q(z) = i\}$, $i \in Q$ where $z \in \Re^l$ is the variable to be quantized. When z does not belong to the union of quantization regions of finite size, the quantizer saturates. More precisely, it is assumed that there exist positive real numbers M and Δ such that the following two conditions hold:

$$|q(z) - z| \leq \Delta, \quad if\ |z| \leq M \quad (6)$$
$$|q(z)| \geq M - \Delta, \quad if\ |z| > M \quad (7)$$

We will refer to M and Δ as the range of $q(z)$ and the quantization error, respectively. Condition 1 in (6) gives a bound on the quantization error when the quantizer does not saturate. Condition 2 in (7) provides a way to detect the possibility of saturation. We also assume that $q(z) = 0$ for z in some neighbourhood of the origin, i.e., the origin lies in the interior of the set $\{z : q(z) = 0\}$, Liberzon (2003); Tian et al. (2008).

In addition, in this paper, we will use quantized measurements of the form

$$q_\mu(z) := \mu q\left(\frac{z}{\mu}\right) = \begin{cases} \mu M \Delta, & \frac{z}{\mu} > (M + 0.5)\Delta \\ -\mu M \Delta, & \frac{z}{\mu} < -(M + 0.5)\Delta \\ \mu \Delta \left[\frac{z}{\mu}\right], & \left|\frac{z}{\mu}\right| \leq (M + 0.5)\Delta \end{cases} \quad (8)$$

where $\mu > 0$ and the range of this quantizer is μM and the quantization error is $\mu \Delta$, Tian et al. (2008).

The problem considered here is to estimate the signal $z(t)$ in (1) by a network-based quantized filter, shown in Fig. 1, of a general structure described by

$$\dot{x}_f(t) = A_f x_f(t) + B_f \mu_{1k} q_1 \left(\frac{y(i_k h)}{\mu_{1k}}\right) \quad (9)$$
$$z_f(t) = C_f x_f(t), \quad t \in [i_k h + \eta_k^{sf}, i_{k+1} h + \eta_{k+1}^{sf}) \quad (10)$$

where $x_f(t)$ is the filter state vector, $\mu_{1k} q_1\left(\frac{y(i_k h)}{\mu_{1k}}\right)$ is the quantized plant output with $i_k h$ as the sampling instant of the sensor and h as the sampling period, $z_f(t)$ is the filter output, and A_f, B_f, C_f are appropriately dimensioned filter matrices to be designed. η_k^{sf} denotes the transmission delay from sensor to the filter. When considering the network conditions from the filter to the plant output, the quantized output signal can be expressed as

$$\mu_{2k} q_2 \left(\frac{z_f(j_k h)}{\mu_{2k}}\right) \quad (11)$$

Define $\eta_1(t) = t - i_k h - \eta_{1m}$ for $t \in [i_k h + \eta_k^{sf}, i_{k+1} h + \eta_{k+1}^{sf})$ and $\eta_2(t) = t - j_k h - \eta_{2m}$ for $t \in [j_k h + \eta_k^{fo}, j_{k+1} h + \eta_{k+1}^{fo})$ with a natural assumption on the network induced delays as follows

$$\eta_{1m} \leq \eta_k^{sf} \leq \eta_{1M} \quad (12)$$
$$\eta_{2m} \leq \eta_k^{fo} \leq \eta_{2M} \quad (13)$$

where constants η_{im} and η_{iM}, $i = 1, 2$, denote the minimum and maximum delays, respectively. η_k^{fo} denotes the transmission delay from the filter to the plant output. Then, from (12)-(13) we have

$$0 \leq \eta_i(t) \leq \bar{\eta}_i \quad (14)$$

where $\bar{\eta}_i := \eta_{iM} - \eta_{im}$. It is assumed that the values in both sets $\{i_1, i_2, i_3, \cdots\}$ and $\{j_1, j_2, j_3, \cdots\}$ are ordered as follows $i_{k+1} > i_k$ and $j_{k+1} > j_k$, which means that there is no wrong packet sequences in the network. Therefore, the following conditions are satisfied, respectively,

$$(i_{k+1} - i_k)h + \eta_k^{sf} < \eta_{1M} \tag{15}$$

$$(j_{k+1} - j_k)h + \eta_k^{fo} < \eta_{2M} \tag{16}$$

Furthermore, it is worth noting that there are $n-1$ continuous packets dropped or lost if $i_{k+1} - i_k = n(n \geq 2)$, Yue and Han (2006).

Replacing $i_k h$ and $j_k h$ in the quantized plant and filter outputs with $t - \eta_{1m} - \eta_1(t)$ and $t - \eta_{2m} - \eta_2(t)$, respectively, in (9) and (10), we obtain

$$\dot{x}_f(t) = A_f x_f(t) + B_f \mu_{1k} q_1 \left(\frac{Cx(t - \eta_{1m} - \eta_1(t))}{\mu_{1k}} \right)$$

$$= A_f x_f(t) + B_f C x(t - \eta_{1m} - \eta_1(t)) + B_f \delta_1(t) \tag{17}$$

and, for $t \in [j_k h + \eta_k^{fo}, j_{k+1} h + \eta_{k+1}^{fo})$,

$$\mu_{2k} q_2 \left(\frac{z_f(j_k h)}{\mu_{2k}} \right) = C_f x_f(t - \eta_{2m} - \eta_2(t)) + \delta_2(t) \tag{18}$$

where

$$\delta_1(t) = \mu_{1k} q_1 \left(\frac{Cx(t - \eta_{1m} - \eta_1(t))}{\mu_{1k}} \right)$$

$$-Cx(t - \eta_{1m} - \eta_1(t)) \tag{19}$$

and

$$\delta_2(t) = \mu_{2k} q_2 \left(\frac{Cx(t - \eta_{2m} - \eta_2(t))}{\mu_{2k}} \right)$$

$$-Cx(t - \eta_{2m} - \eta_2(t)) \tag{20}$$

By connecting the plant (1)-(3) and the filter (9)-(10) and from the Leibniz-Newton formula, i.e.

$$X(t - \eta_{1m} - \eta_1(t)) = X(t - \eta_{1m})$$

$$- \int_{t-\eta_{1m}-\eta_1(t)}^{t-\eta_{1m}} \dot{X}(s)\, ds \tag{21}$$

we obtain the filtering error system as

$$\dot{X}(t) = (\bar{A} + \Delta\bar{A}(t))X(t) + \bar{B}_1 X(t - \eta_{1m})$$

$$-\bar{B}_1 \int_{t-\eta_{1m}-\eta_1(t)}^{t-\eta_{1m}} \dot{X}(s)\, ds + \bar{B}_2 \delta_1(t)$$

$$+(\bar{B}_3 + \Delta\bar{B}(t))w(t) \tag{22}$$

and

$$e(t) = z(t) - \mu_{2k} q_2 \left(\frac{z_f(j_k h)}{\mu_{2k}} \right)$$

$$= \bar{C}_1 X(t) + \bar{C}_2 X(t - \eta_{2m} - \eta_2(t)) - \delta_2(t)$$

$$= \bar{C}_1 X(t) + \bar{C}_2 X(t - \eta_{2m})$$

$$-\bar{C}_2 \int_{t-\eta_{2m}-\eta_2(t)}^{t-\eta_{2m}} \dot{X}(s)\, ds - \delta_2(t) \tag{23}$$

where

$$\bar{A} = \begin{bmatrix} A & 0 \\ 0 & A_f \end{bmatrix}, \Delta\bar{A}(t) = \begin{bmatrix} \Delta A(t) & 0 \\ 0 & 0 \end{bmatrix},$$

$$\bar{B}_1 = \begin{bmatrix} 0 & 0 \\ B_f C & 0 \end{bmatrix}, \bar{B}_2 = \begin{bmatrix} 0 \\ B_f \end{bmatrix}, \bar{B}_3 = \begin{bmatrix} B \\ 0 \end{bmatrix},$$

$$\Delta\bar{B}_3 = \begin{bmatrix} \Delta B(t) \\ 0 \end{bmatrix}, \bar{C}_1 = \begin{bmatrix} G & 0 \end{bmatrix}, \bar{C}_2 = \begin{bmatrix} 0 & -C_f \end{bmatrix}$$

Finally, the problem of robust H_∞ filtering for uncertain linear systems with both the output quantization levels and the network conditions can be expressed as below.

Problem: Given system (1)-(3), design the filter (9)-(10) such that the filtering error system (22)-(23) from $w(t)$ to $e(t)$ is asymptotically stable with a prescribed H_∞ performance γ, that is $\|e(t)\|_2^2 < \gamma^2 \|w(t)\|_2^2$ under zero initial conditions for all admissible uncertain parameters.

Remark 1. It can be easily seen that the model under consideration in this paper is different from existing results in Yue and Han (2006) and Yue et al. (2006) in the following perspective: in comparison with our case that the filtering error system in (22) also considers the network conditions from the filter to the plant output, i.e. the quantized controlled output signal in (11), the references Yue and Han (2006); Yue et al. (2006) do not center on this case, i.e., the results in Yue and Han (2006) and Yue et al. (2006) can not be directly applied to the system (22)-(23).

3 H_∞ performance analysis

In this section, we first investigate the problem of H_∞ performance analysis for nominal system (1)-(3) with no uncertainties and exactly known filter matrices. Specifically, we will be concerned with the conditions under which the filtering error system with finite delay components is asymptotically stable from $w(t)$ to $e(t)$ with an H_∞ performance γ. The following theorem shows that the H_∞ performance of the filtering error system can be guaranteed if one can find certain matrices so that some LMIs are satisfied.

Theorem 1. Given the positive constants γ, Δ_i and the matrices A_f, B_f, C_f, if there exist positive-definite matrices $P_1, R_1, R_2, S_1, S_2, Q_1, Q_2, Z_1, Z_2, T_1, T_2$ and matrices $H, U, N_{i,j} (i = 1, 2, \cdots, 4; j = 1, 2, \cdots, 10)$ of appropriate dimensions such that the following LMIs hold

$$\begin{bmatrix} \Pi & \eta_{1m}\chi_1 & \eta_{2m}\chi_2 & \eta_{1M}\chi_3 & \eta_{2M}\chi_4 \\ * & -\eta_{1m}T_1 & 0 & 0 & 0 \\ * & * & -\eta_{2m}T_2 & 0 & 0 \\ * & * & * & -\eta_{1M}Q_1 & 0 \\ * & * & * & * & -\eta_{2M}Q_2 \end{bmatrix} < 0 \tag{24}$$

$$\begin{bmatrix} H & U \\ * & Z_1 \end{bmatrix} \geq 0 \tag{25}$$

with $\chi_i = [N_{i,1}^T, N_{i,2}^T, \cdots, N_{i,10}^T, 0]^T$ $(i = 1, 2, \cdots, 4)$, $\Pi = \Pi^T = [\Pi_{i,j}]_{i,j=1,2,\cdots,11}$, $\tilde{N}_i = N_{1,i} + N_{2,i} + N_{3,i} + N_{4,i}$, $P = \begin{bmatrix} P_1 & 0 \\ P_3 & P_2 \end{bmatrix}$ and $\Pi_{1,1} = sym\left\{P^T \begin{bmatrix} 0 & I \\ \bar{A} & -I \end{bmatrix}\right\} + \bar{\eta}_1 H + diag\left\{R_1 + S_1, \sum_{i=1}^{2} \eta_{iM}Q_i + \bar{\eta}_i Z_i + \eta_{im}T_i\right\} + sym\left\{\tilde{N}_1 \hat{I}\right\}$, $\Pi_{1,2} = P^T \begin{bmatrix} 0 \\ \bar{B}_1 \end{bmatrix} - N_{1,1} + \hat{I}\tilde{N}_2^T$, $\Pi_{1,3} = U - P^T \begin{bmatrix} 0 \\ \bar{B}_1 \end{bmatrix} - N_{1,1} + \hat{I}\tilde{N}_3^T$, $\Pi_{1,4} = -N_{3,1} + \hat{I}\tilde{N}_4^T$, $\Pi_{1,5} = -N_{2,1} + \hat{I}\tilde{N}_5^T$, $\Pi_{1,6} = -N_{4,1} + \hat{I}\tilde{N}_6^T$, $\Pi_{1,7} = \hat{I}\tilde{N}_7^T$, $\Pi_{1,8} = P^T \begin{bmatrix} 0 \\ \bar{B}_2 \end{bmatrix} + \hat{I}\tilde{N}_8^T$, $\Pi_{1,9} = \hat{I}\tilde{N}_9^T$, $\Pi_{1,10} = P^T \begin{bmatrix} 0 \\ \bar{B}_3 \end{bmatrix} + \hat{I}\tilde{N}_{10}^T$, $\Pi_{1,11} = \begin{bmatrix} \bar{C}_1 & 0 \end{bmatrix}^T$, $\Pi_{2,2} = -R_1 - R_2 - sym\{N_{1,2}\}$, $\Pi_{2,3} = -\frac{\Delta_1^2}{M_1^2\mu_{1k}^2}\hat{I}^T C^T C \hat{I} - N_{1,3}^T$, $\Pi_{2,4} = -N_{1,2} - N_{1,4}^T$, $\Pi_{2,6} = -N_{4,2} - N_{1,6}^T$, $\Pi_{2,7} = -N_{1,7}^T$, $\Pi_{2,8} = -N_{1,8}^T$, $\Pi_{2,9} = -N_{1,9}^T$, $\Pi_{2,10} = -N_{1,10}^T$, $\Pi_{3,3} = -\bar{\eta}_1^{-1} Z_1 + \frac{\Delta_1^2}{M_1^2\mu_{1k}^2}\hat{I}^T C^T C \hat{I}$, $\Pi_{3,4} = -N_{3,3}$, $\Pi_{3,5} = -N_{2,3}$, $\Pi_{3,6} = -N_{4,3}$, $\Pi_{4,4} = -R_2 - sym\{N_{3,4}\}$, $\Pi_{4,5} = -N_{2,4} - N_{3,5}^T$, $\Pi_{4,6} = -N_{4,4} - N_{3,6}^T$, $\Pi_{4,7} = -N_{3,7}^T$, $\Pi_{4,8} = -N_{3,8}^T$, $\Pi_{4,9} = -N_{3,9}^T$, $\Pi_{4,10} = -N_{3,10}^T$, $\Pi_{5,5} = S_2 - S_1 - sym\{N_{2,5}\} + \frac{\Delta_2^2}{M_2^2\mu_{2k}^2}\tilde{I}^T C_f^T C_f \tilde{I}$, $\Pi_{5,6} = -N_{4,5} - N_{2,6}^T$, $\Pi_{5,7} = -N_{2,7}^T - \frac{\Delta_2^2}{M_2^2\mu_{2k}^2}\tilde{I}^T C_f^T C_f \tilde{I}$, $\Pi_{5,8} = -N_{2,8}^T$, $\Pi_{5,9} = -N_{2,9}^T$, $\Pi_{5,10} = -N_{2,10}^T$, $\Pi_{5,11} = \bar{C}_2^T$, $\Pi_{6,6} = -S_2 - sym\{N_{4,6}\}$, $\Pi_{6,7} = -N_{4,7}^T$, $\Pi_{6,8} = -N_{4,8}^T$, $\Pi_{6,9} = -N_{4,9}^T$, $\Pi_{6,10} = -N_{4,10}^T$, $\Pi_{7,7} = -\bar{\eta}_2^{-1} Z_2 + \frac{\Delta_2^2}{M_2^2\mu_{2k}^2}\tilde{I}^T C_f^T C_f \tilde{I}$, $\Pi_{7,11} = -\bar{C}_2^T$, $\Pi_{8,8} = \Pi_{9,9} = -I$, $\Pi_{9,11} = -I$, $\Pi_{10,10} = -\gamma^2 I$, $\Pi_{11,11} = -I$ and other elements $\Pi_{i,j}$ for $j \geq i$ are equal to zero. Then, system (22)-(23) is asymptotically stable with the H_∞ performance level $\gamma > 0$.

Proof. Firstly, we represent (22) in an equivalent descriptor model form as

$$\begin{cases} \dot{X}(t) = \xi(t), \\ 0 = -\xi(t) + \bar{A}X(t) + \bar{B}_1 X(t - \eta_{1m}) \\ -\bar{B}_1 \int_{t-\eta_{1m}-\eta_1(t)}^{t-\eta_{1m}} \dot{X}(s)\, ds + \bar{B}_2 \delta_1(t) + \bar{B}_3 w(t) \end{cases} \tag{26}$$

Define the Lyapunov-Krasovskii functional

$$V(t) = \sum_{i=1}^{5} V_i(t) \tag{27}$$

where

$$V_1(t) = X(t)^T P_1 X(t) := \begin{bmatrix} X(t)^T & \xi(t)^T \end{bmatrix} TP \begin{bmatrix} X(t) \\ \xi(t) \end{bmatrix},$$

$$V_2(t) = \int_{t-\eta_{1m}}^{t} X(s)^T R_1 X(s)\, ds + \int_{t-\eta_{1M}}^{t-\eta_{1m}} X(s)^T R_2 X(s)\, ds$$

$$V_3(t) = \int_{t-\eta_{2m}}^{t} X(s)^T S_1 X(s)\, ds + \int_{t-\eta_{2M}}^{t-\eta_{2m}} X(s)^T S_2 X(s)\, ds$$

$$V_4(t) = \int_{-\eta_{1M}}^{0} \int_{t+\theta}^{t} \xi(s)^T Q_1 \xi(s)\, ds\, d\theta + \int_{-\eta_{1M}}^{-\eta_{1m}} \int_{t+\theta}^{t} \xi(s)^T Z_1 \xi(s)\, ds\, d\theta + \int_{-\eta_{1m}}^{0} \int_{t+\theta}^{t} \xi(s)^T T_1 \xi(s)\, ds\, d\theta$$

$$V_5(t) = \int_{-\eta_{2M}}^{0} \int_{t+\theta}^{t} \xi(s)^T Q_2 \xi(s)\, ds\, d\theta + \int_{-\eta_{2M}}^{-\eta_{2m}} \int_{t+\theta}^{t} \xi(s)^T Z_2 \xi(s)\, ds\, d\theta + \int_{-\eta_{2m}}^{0} \int_{t+\theta}^{t} \xi(s)^T T_2 \xi(s)\, ds\, d\theta$$

with $T = diag\{I, 0\}$. Differentiating $V_1(t)$ in t we obtain

$$\dot{V}_1(t) = 2X(t)^T P_1 \dot{X}(t) = 2\begin{bmatrix} X(t)^T & \xi(t)^T \end{bmatrix} P^T \begin{bmatrix} \dot{X}(t) \\ 0 \end{bmatrix}$$

$$= 2\begin{bmatrix} X(t)^T & \xi(t)^T \end{bmatrix} P^T \left\{ \begin{bmatrix} 0 & I \\ \bar{A} & -I \end{bmatrix} \begin{bmatrix} X(t) \\ \xi(t) \end{bmatrix} \right.$$

$$+ \begin{bmatrix} 0 \\ \bar{B}_1 \end{bmatrix} X(t-\eta_{1m}) - \begin{bmatrix} 0 \\ \bar{B}_1 \end{bmatrix} \int_{t-\eta_{1m}-\eta_1(t)}^{t-\eta_{1m}} \xi(s)\ ds$$

$$+ \begin{bmatrix} 0 \\ \bar{B}_2 \end{bmatrix} \delta_1(t) + \begin{bmatrix} 0 \\ \bar{B}_3 \end{bmatrix} w(t)\} \tag{28}$$

By Lemma 1 (in Appendix) and from (25), it is clear that

$$-2\begin{bmatrix} X(t)^T & \xi(t)^T \end{bmatrix} P^T \begin{bmatrix} 0 \\ \bar{B}_1 \end{bmatrix} \int_{t-\eta_{1m}-\eta_1(t)}^{t-\eta_{1m}} \xi(s)\ ds$$

$$\leq \int_{t-\eta_{1m}-\eta_1(t)}^{t-\eta_{1m}} \begin{bmatrix} X(t) \\ \xi(t) \\ \xi(s) \end{bmatrix}^T \begin{bmatrix} H & U - P^T \begin{bmatrix} 0 \\ \bar{B}_1 \end{bmatrix} \\ * & Z_1 \end{bmatrix} \begin{bmatrix} X(t) \\ \xi(t) \\ \xi(s) \end{bmatrix} ds$$

$$\leq \int_{t-\eta_{1M}}^{t-\eta_{1m}} \xi(s)^T Z_1 \xi(s)\ ds + \bar{\eta}_1 \begin{bmatrix} X(t) \\ \xi(t) \end{bmatrix}^T H \begin{bmatrix} X(t) \\ \xi(t) \end{bmatrix}$$

$$+2\begin{bmatrix} X(t) \\ \xi(t) \end{bmatrix}^T (U - P^T \begin{bmatrix} 0 \\ \bar{B}_1 \end{bmatrix}) \int_{t-\eta_{1m}-\eta_1(t)}^{t-\eta_{1m}} \xi(s)\ ds \tag{29}$$

Differentiating other Lyapunov terms in (27) give

$$\dot{V}_2(t) + \dot{V}_3(t) = X(t)^T (R_1 + S_1) X(t) - X(t-\eta_{1m})^T$$

$$\times (R_1 - R_2) X(t-\eta_{1m}) - X(t-\eta_{2M})^T S_2 X(t-\eta_{2M})$$

$$-X(t-\eta_{1M})^T R_2 X(t-\eta_{1M})$$

$$-X(t-\eta_{2m})^T (S_1 - S_2) X(t-\eta_{2m}) \tag{30}$$

and, using Jensen's Inequality in Lemma 2 (in Appendix), one gets

$$\dot{V}_4(t) = \xi(t)^T (\eta_{1M} Q_1 + \bar{\eta}_1 Z_1 + \eta_{1m} T_1) \xi(t)$$

$$-\int_{t-\eta_{1M}}^{t} \xi(s)^T Q_1 \xi(s)\ ds - \int_{t-\eta_{1M}}^{t-\eta_{1m}} \xi(s)^T Z_1 \xi(s)\ ds$$

$$-\int_{t-\eta_{1m}}^{t} \xi(s)^T T_1 \xi(s)\ ds$$

$$\leq \xi(t)^T (\eta_{1M} Q_1 + \bar{\eta}_1 Z_1 + \eta_{1m} T_1) \xi(t)$$

$$-\int_{t-\eta_{1M}}^{t} \xi(s)^T Q_1 \xi(s)\ ds - \int_{t-\eta_{1m}-\eta_1(t)}^{t-\eta_{1m}} \xi(s)^T Z_1 \xi(s)\ ds$$

$$-\int_{t-\eta_{1m}}^{t} \xi(s)^T T_1 \xi(s)\ ds$$

$$\leq \xi(t)^T (\eta_{1M} Q_1 + \bar{\eta}_1 Z_1 + \eta_{1m} T_1) \xi(t)$$

$$-\int_{t-\eta_{1M}}^{t} \xi(s)^T Q_1 \xi(s)\ ds$$

$$-\bar{\eta}_1^{-1} (\int_{t-\eta_{1m}-\eta_1(t)}^{t-\eta_{1m}} \xi(s)^T\ ds) Z_1 (\int_{t-\eta_{1m}-\eta_1(t)}^{t-\eta_{1m}} \xi(s)\ ds)$$

$$-\int_{t-\eta_{1m}}^{t} \xi(s)^T T_1 \xi(s)\ ds \tag{31}$$

and, similarly,

$$\dot{V}_5(t) \leq \xi(t)^T (\eta_{2M} Q_2 + \bar{\eta}_2 Z_2 + \eta_{2m} T_2) \xi(t)$$

$$-\int_{t-\eta_{2M}}^{t} \xi(s)^T Q_2 \xi(s)\ ds$$

$$-\bar{\eta}_2^{-1} (\int_{t-\eta_{2m}-\eta_2(t)}^{t-\eta_{2m}} \xi(s)^T\ ds) Z_2 (\int_{t-\eta_{2m}-\eta_2(t)}^{t-\eta_{2m}} \xi(s)\ ds)$$

$$-\int_{t-\eta_{2m}}^{t} \xi(s)^T T_2 \xi(s)\ ds \tag{32}$$

Moreover, from the Leibniz-Newton formula, the following equations hold for any matrices $\{N_i\}_{i=1}^{10}$ with appropriate dimensions:

$$2\nu(t)^T T_1 (X(t) - X(t-\eta_{1m}) - \int_{t-\eta_{1m}}^{t} \xi(s)\ ds) = 0 \tag{33}$$

$$2\nu(t)^T T_2 (X(t) - X(t-\eta_{2m}) - \int_{t-\eta_{2m}}^{t} \xi(s)\ ds) = 0 \tag{34}$$

$$2\nu(t)^T T_3 (X(t) - X(t-\eta_{1M}) - \int_{t-\eta_{1M}}^{t} \xi(s)\ ds) = 0 \tag{35}$$

$$2\nu(t)^T T_4(X(t) - X(t-\eta_{2M}) - \int_{t-\eta_{2M}}^{t} \xi(s)\, ds) = 0 \tag{36}$$

where $\nu(t) := col\{X(t), \xi(t), X(t-\eta_{1m}), \int_{t-\eta_{1m}-\eta_1(t)}^{t-\eta_{1m}} \xi(s)\, ds, X(t-\eta_{1M}), X(t-\eta_{2m}), X(t-\eta_{2M}), \int_{t-\eta_{2m}-\eta_2(t)}^{t-\eta_{2m}} \xi(s)\, ds, \delta_1(t), \delta_2(t), w(t)\}$ is an augmented state vector. According to the property of the quantizers $q_i(.)$ and using the Leibniz-Newton formula, we readily obtain

$$\begin{aligned} 0 &\le -\delta_1(t)^T\delta_1(t) \\ &+ \frac{\Delta_1^2}{M_1^2\mu_{1k}^2} x(t-\eta_{1m}-\eta_1(t))^T C^T C x(t-\eta_{1m}-\eta_1(t)) \\ &= -\delta_1(t)^T\delta_1(t) + \frac{\Delta_1^2}{M_1^2\mu_{1k}^2}(x(t-\eta_{1m}) \\ &- \int_{t-\eta_{1m}-\eta_1(t)}^{t-\eta_{1m}} \dot{x}(s)\, ds)^T C^T C(x(t-\eta_{1m}) \\ &- \int_{t-\eta_{1m}-\eta_1(t)}^{t-\eta_{1m}} \dot{x}(s)\, ds) \end{aligned} \tag{37}$$

and, similarly,

$$\begin{aligned} 0 &\le -\delta_2(t)^T\delta_2(t) + \frac{\Delta_2^2}{M_2^2\mu_{2k}^2}(x(t-\eta_{2m}) \\ &- \int_{t-\eta_{2m}-\eta_2(t)}^{t-\eta_{2m}} \dot{x}(s)\, ds)^T C^T C(x(t-\eta_{2m}) \\ &- \int_{t-\eta_{2m}-\eta_2(t)}^{t-\eta_{2m}} \dot{x}(s)\, ds) \end{aligned} \tag{38}$$

Now, to establish the H_∞ performance measure for the system (1)-(3), assume zero initial condition, then we have $V(t)\,|_{t=0} = 0$. Consider the index J_∞ in the form $J_\infty = \int_0^\infty [e(t)^T e(t) - \gamma^2 w(t)^T w(t)]\, dt$, then along the solution of (1) for any nonzero $w(t)$ the following equation holds

$$\begin{aligned} J_\infty &\le \int_0^\infty [e(t)^T e(t) - \gamma^2 w(t)^T w(t)]\, dt \\ &- V(t)\,|_{t=0} + V(t)|_{t=\infty} \\ &= \int_0^\infty [e(t)^T e(t) - \gamma^2 w(t)^T w(t) + \dot{V}(t)]\, dt \end{aligned} \tag{39}$$

From (19), (28)-(32) and adding the left and right sides of equations (33)-(36) and (37)-(38), respectively, into $\dot{V}(t)$, we get

$$\begin{aligned} J_\infty &\le \int_0^\infty \nu(t)^T\Sigma\nu(t)\, dt - \int_0^\infty \int_{t-\eta_{1m}}^{t} (\nu(t)^T\chi_1 \\ &+ \xi(s)^T T_1)T_1^{-1}(\nu(t)^T\chi_1 + \xi(s)^T T_1)^T\, ds\, dt \\ &- \int_0^\infty \int_{t-\eta_{2m}}^{t} (\nu(t)^T\chi_2 + \xi(s)^T T_2)T_2^{-1}(\nu(t)^T\chi_2 \\ &+ \xi(s)^T T_2)^T\, ds\, dt - \int_0^\infty \int_{t-\eta_{1M}}^{t} (\nu(t)^T\chi_3 + \xi(s)^T Q_1)Q_1^{-1} \\ &\times(\nu(t)^T\chi_3 + \xi(s)^T Q_1)^T\, ds\, dt - \int_0^\infty \int_{t-\eta_{2M}}^{t} (\nu(t)^T\chi_4 \\ &+ \xi(s)^T Q_2)Q_2^{-1}(\nu(t)^T\chi_4 + \xi(s)^T Q_2)^T\, ds\, dt \end{aligned} \tag{40}$$

where $\Sigma := \Pi + \eta_{1M}\chi_1 T_1^{-1}\chi_1^T + \eta_{2M}\chi_2 T_2^{-1}\chi_2^T + \eta_{1M}\chi_3 Q_1^{-1}\chi_3^T + \eta_{2M}\chi_4 Q_2^{-1}\chi_4^T$. Now, if $\Sigma < 0$, then $J_\infty < 0$ which means that the L_2-gain from the disturbance $w(t)$ to the filtering error $e(t)$ is less than γ. By applying Schur complements, we find that $\Sigma < 0$ is equivalent to (24). This completes the proof. $\triangleleft$

4 Robust H_∞ filter design

In this section we investigate the robust H_∞ filter design problem for system (1)-(3) with the norm bounded uncertainty parameters defined in (4)-(5).

Theorem 2. Consider system (1)-(3) with the quantizer given in (8). Given positive constants ϵ, γ and Δ_i, there exist a network-based quantized filter in the form of (9)-(10) such that the filtering error system (22)-(23) is asymptotically stable with an H_∞ disturbance attenuation level γ if there exist the scalar $\rho > 0$, positive-definite matrices $P_1, R_1, R_2, S_1, S_2, Q_1, Q_2, Z_1, Z_2, T_1, T_2$ and matrices $C_f, W_1, W_2, H, U, N_{i,j}(i = 1, 2, \cdots, 4; j = 1, 2, \cdots, 10)$ of appropriate dimensions and satisfying (25) and the LMI

$$\begin{bmatrix} \tilde{\Pi} & \Gamma_d^T & \rho\Gamma_e^T \\ * & -\rho I & 0 \\ * & * & -\rho I \end{bmatrix} < 0 \tag{41}$$

with

$$\tilde{\Pi} := \begin{bmatrix} \hat{\Pi} & \eta_{1m}\chi_1 & \eta_{2m}\chi_2 & \eta_{1M}\chi_3 & \eta_{2M}\chi_4 \\ * & -\eta_{1m}T_1 & 0 & 0 & 0 \\ * & * & -\eta_{2m}T_2 & 0 & 0 \\ * & * & * & -\eta_{1M}Q_1 & 0 \\ * & * & * & * & -\eta_{2M}Q_2 \end{bmatrix}$$

$\Gamma_d = \begin{bmatrix}\epsilon M_1^T P_2 & M_1^T P_2 & \cdots & 0 & 0\end{bmatrix}$, $\Gamma_e = \begin{bmatrix}L_a & 0 & \cdots & 0 & L_b & 0\end{bmatrix}$ with $\hat{\Pi} = \hat{\Pi}^T = [\hat{\Pi}_{i,j}]_{i,j=1,2,\cdots,11}$, and $P_2 = \begin{bmatrix}P_{11} & P_{12} \\ P_{22} & P_{22}\end{bmatrix}$ and

$$\hat{\Pi}_{1,1} := sym\{\begin{bmatrix}\epsilon\begin{bmatrix}P_{11}^T A & W_1 \\ P_{12}^T A & W_1\end{bmatrix} & P_1 - \epsilon P_2^T \\ \begin{bmatrix}P_{11}^T A & W_1 \\ P_{12}^T A & W_1\end{bmatrix} & -P_2^T\end{bmatrix}\}$$
$$+\bar{\eta}_1 H + diag\left\{R_1 + S_1, \sum_{i=1}^{2} \eta_{iM} Q_i + \bar{\eta}_i Z_i + \eta_{im} T_i\right\}$$
$$+sym\left\{\tilde{N}_1 \hat{I}\right\},$$

$\hat{\Pi}_{1,2} = (\epsilon\hat{I}^T + \tilde{I}^T)\begin{bmatrix}W_2 C & 0 \\ W_2 C & 0\end{bmatrix} - N_{1,1} + \hat{I}^T\tilde{N}_2^T$, $\hat{\Pi}_{1,3} = U - (\epsilon\hat{I}^T + \tilde{I}^T)\begin{bmatrix}W_2 C & 0 \\ W_2 C & 0\end{bmatrix} + \hat{I}^T\tilde{N}_3^T$, $\hat{\Pi}_{1,8} = (\epsilon\hat{I}^T + \tilde{I}^T)\begin{bmatrix}W_2 \\ W_2\end{bmatrix} + \hat{I}^T\tilde{N}_8^T$, $\hat{\Pi}_{1,10} = (\epsilon\hat{I}^T + \tilde{I}^T)\begin{bmatrix}P_{11}^T B \\ P_{22}^T B\end{bmatrix} + \hat{I}^T\tilde{N}_{10}^T$ and other elements $\hat{\Pi}_{i,j}$ are equal to their counterpart elements in the matrix Π. Moreover, if the above conditions are feasible, desired filter gain matrices in the form of (9)-(10) are given by C_f and

$$\begin{bmatrix}A_f & B_f\end{bmatrix} = (P_{22}^T)^{-1}\begin{bmatrix}W_1 & W_2\end{bmatrix} \tag{42}$$

Proof. If the state-space matrices $\bar{A}$ and $\bar{B}_3$ in (24) are replaced with $\bar{A} + M_1 F(t) L_a$ and $\bar{B}_3 + M_1 F(t) L_b$, respectively, and by considering $P_3 = \epsilon P_2$ and introducing change of variables

$$\begin{bmatrix}W_1 & W_2\end{bmatrix} = P_{22}^T\begin{bmatrix}A_f & B_f\end{bmatrix} \tag{43}$$

then the inequality (24) is equivalent to the following condition:

$$\tilde{\Pi} + sym\left\{\Gamma_d^T F(t)\Gamma_e\right\} < 0 \tag{44}$$

By Lemma 3 (in Appendix), a necessary and sufficient condition for (44) is that there exists a scalar ρ such that

$$\tilde{\Pi} + \rho^{-1}\Gamma_d^T\Gamma_d + \rho\Gamma_e^T\Gamma_e < 0 \tag{45}$$

then, applying Schur complements, we find that (45) is equivalent to (41). This completes the proof. ◁

Remark 2. In Theorem 2, the results are expressed within the framework of LMIs, which can be easily computed by the interior-pint method. It is also observed that the LMIs (25) and (41) are linear in the set of scalar ρ, matrices $P_1, R_1, R_2, S_1, S_2, Q_1, Q_2, Z_1, Z_2, T_1, T_2, C_f, W_1, W_2, H, U, N_{i,j}(i = 1, 2, \cdots, 4; j = 1, 2, \cdots, 10)$.

Then, the optimal solution to obtain the minimum disturbance attenuation level, i.e. γ, can be found by solving the following convex optimization problem

$$min \quad \lambda$$

$$subject\ to\ LMIs\ (25)\ and\ (41)\ with\ \lambda := \gamma^2$$

Remark 3. The reduced conservatism of Theorems 1 and 2 benefits from the construction of the new Lyapunov function in (27), using a free weighting matrix technique, and no bounding technique is needed to estimate the inner product of the involved crossing terms Park (1999). It is also worth noting that, recently, the so-called 'delay fractioning' approach has been developed in Mou et al. (2008) that is shown to lead to much less conservative results than most existing literature. Of course, more detailed investigations using delay fractioning method would be of interest.

5 Numerical results

In this section, one example is provided to illustrate the effectiveness of the results obtained in the previous sections.

Consider the system (1)-(3) with the following matrices:

$$A = \begin{bmatrix}-1 & 0 \\ 1 & -3\end{bmatrix}, B = diag\{0.2, 0.2\},$$
$$C = \begin{bmatrix}1 & 0\end{bmatrix}, G = \begin{bmatrix}0.2 & 0.3\end{bmatrix},$$
$$L_a = \begin{bmatrix}0 & 0 \\ 0.2 & 0.2\end{bmatrix}, L_b = \begin{bmatrix}0 & 0 \\ 0.1 & 0.1\end{bmatrix}, M_1 = I$$

By applying Theorem 2 to the system under consideration with constants $\epsilon = 0.1, \eta_{1m} = \eta_{2m} = 0, \eta_{1M} = \eta_{2M} = 1, \Delta_1 = \Delta_2 = 0.1$ and disturbance attenuation level $\gamma = 1$, one can obtain the network-based quantized filter (6) with the following state-space matrices

$$A_f = \begin{bmatrix}-0.7856 & -0.1322 \\ -0.0295 & -0.6554\end{bmatrix}, \quad B_f = \begin{bmatrix}0.2161 \\ 0.0184\end{bmatrix}$$

$$C_f = \begin{bmatrix}0.1085 & 0.1655\end{bmatrix}.$$

For the quantizer parameters $\Delta_1 = \Delta_2 = 0.1, M_1 = M_2 = 5$ in (5) with the initial conditions $x(0) = \begin{bmatrix}-1 & 0.2\end{bmatrix}^T$ and $x_f(0) = \begin{bmatrix}0 & 0\end{bmatrix}^T$, the delays $\eta_1(t) = \eta_2(t) = (1 - e^{-t})/(1 + e^{-t})$ and exogenous disturbance inputs as below, which belongs to $[0, \infty]$,

$$w(t) = \begin{bmatrix}1/(1 + 2t^2) \\ 1/(1 + 0.5t^{0.5})\end{bmatrix}, \quad t \geq 0$$

then the filtering error signal $e(t)$ is plotted in Fig. 2 which shows that the controlled output of the filter, $z_f(t)$, tracks the controlled output of the system, $z(t)$, well.

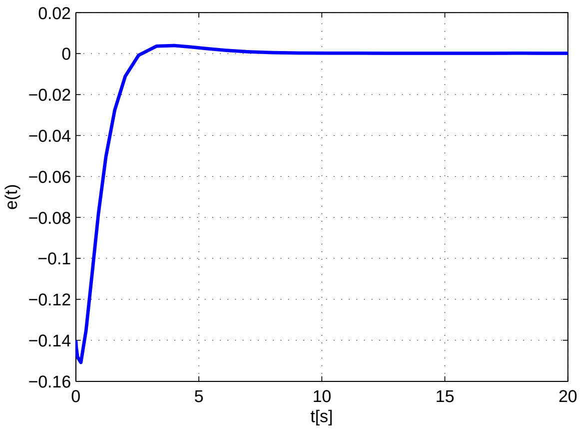

Figure 2: Filtering error signal.

6 Conclusion

In this paper we have developed the problem of robust H_∞ filtering for uncertain linear systems connected over a common digital communication network. We considered the case where quantizers are static and the parameter uncertainties are norm bounded. Firstly, we proposed a new model to investigate the effect of both the output quantization levels and the network conditions. Secondly, by introducing a descriptor technique, using Lyapunov-Krasovskii functional and a suitable change of variables, new required sufficient conditions were established in terms of delay-dependent linear matrix inequalities (LMIs) for the existence of the desired network-based quantized filters with simultaneous consideration of network induced delays and measurement quantization. The explicit expression of the filters was derived to satisfy both asymptotic stability and a prescribed level of disturbance attenuation for all admissible norm bounded uncertainties. Future work will investigate filter and control designs for a class of nonlinear systems over network with random communication packet losses by using the delay fractioning approach.

Appendix

Lemma 1 Han and Yu (2004): For any arbitrary column vectors $a(t), b(t)$, matrices $\Phi(t), H, U$ and W the following inequality holds:

$$-2\int_{t-r}^{t} a(s)^T \Phi(s) b(s)\ ds \leq$$

$$\int_{t-r}^{t} \begin{bmatrix} a(s) \\ b(s) \end{bmatrix}^T \begin{bmatrix} H & U-\Phi(s) \\ * & W \end{bmatrix} \begin{bmatrix} a(s) \\ b(s) \end{bmatrix}\ ds$$

where $\begin{bmatrix} H & U \\ * & W \end{bmatrix} \geq 0$.

Lemma 2: Park (1999) (*Jensen's Inequality*) Given a positive-definite matrix $P \in \Re^{n\times n}$ and two scalars $b > a \geq 0$ for any vector $x(t) \in \Re^n$, we have

$$\int_{t-b}^{t-a} x(w)^T P x(w)\ dw \geq$$

$$\frac{1}{b-a}\left(\int_{t-b}^{t-a} x(w)\ dw\right)^T P \left(\int_{t-b}^{t-a} x(w)\ dw\right)$$

Lemma 3 Khargonekar et al. (1990): Given matrices $Y = Y^T, D, E$ and F of appropriate dimensions with $F^T F \leq I$, then the following matrix inequality

$$Y + sym\{DFE\} < 0$$

holds for all F if and only if there exists a scalar $\epsilon > 0$ such that

$$Y + \epsilon DD^T + \epsilon^{-1} E^T E < 0.$$

References

Branicky, M., Phillips, S., and Zhang, W. Stability of networked control systems: explicit analysis of delay. In *Proc. American Control Conference*. Chicago, Illinois, USA, 2000 pages 2352–2357.

Gao, H. and Chen, T. H_∞ estimation for uncertain systems with limited communication capacity. *IEEE Trans. on Automatic Control*, 2007. 52(11):2070–2084. doi:10.1109/TAC.2007.908316.

Gao, H. and Chen, T. Network based H_∞ output tracking control. *IEEE Trans. on Automatic Control*, 2008. 53(3):655–667. doi:10.1109/TAC.2008.919850.

Gao, H., Chen, T., and Lam, J. A new delay system approach to network-based control. *Automatica*, 2008. 44(1):39–52. doi:10.1016/j.automatica.2007.04.020.

Gao, H., Shi, P., and Wang, J. Parameter-dependent robust stability of uncertain time-delay systems. *J. Computational and Applied Mathematics*, 2007. 206(1):366–373. doi:10.1016/j.cam.2006.07.027.

Gao, H. and Wang, C. Comments and further results on 'a descriptor system approach to H_∞ control of linear time-delay systems.'. *IEEE Trans. on Automatic Control*, 2003. 48(3):520–525. doi:10.1109/TAC.2003.809154.

Han, Q. and Yu, L. Robust stability of linear neutral systems with nonlinear parameter perturbations. *IEE Proc. Control Theory Appl.*, 2004. 151(5):539–546. doi:10.1049/ip-cta:20040785.

Hu, S. and Zhu, Q. Stochastic optimal control and analysis of stability of networked control systems with long delay. *Automatica*, 2003. 39(11):1877–1884. doi:10.1016/S0005-1098(03)00196-1.

Ishii, H. and Francis, B. *Limited data rate in control systems with networks*. Springer-Verlag, Berlin, 2002.

Karimi, H. and Gao, H. LMI-based delay-dependent mixed H_2/H_∞ control of second-order neutral systems with time-varying state and input delays. *ISA Transactions*, 2008. 47(3):311–324. doi:10.1016/j.isatra.2008.04.002.

Karimi, H. and Gao, H. New delay-dependent exponential H_∞ synchronization for uncertain neural networks with mixed time delays. *IEEE Trans. Systems, Man, Cybernetics-Part B*, 2009a. (99):1877–1884. doi:10.1109/TSMCB.2009.2024408.

Karimi, H. and Gao, H. LMI-based H_∞ synchronization of second-order neutral master-slave systems using delayed output feedback control. *Int. J. Control, Automation, and Systems*, 2009b. 7(3):371–380. doi:10.1007/s12555-009-0306-5.

Karimi, H. and Maass, P. Delay-range-dependent exponential H_∞ synchronization of a class of delayed neural networks. *Chaos, Solitons & Fractals*, 2009. 41(3):1125–1135. doi:10.1016/j.chaos.2008.04.051.

Karimi, H., Zapateiro, M., and Luo, N. Robust mixed H_2/H_∞ delayed state-feedback control of neutral delay systems with time-varying delays. *Asian J. of Control*, 2008. 10(5):571–582. doi:10.1002/asjc.57.

Khargonekar, P., Petersen, I., and Zhou, K. Robust stabilization of uncertain linear systems: Quadratic stabilizability and H_∞ control theory. *IEEE Trans. on Automatic Control*, 1990. 35(3):356–361. doi:10.1109/9.50357.

Kim, D., Lee, Y., Kwon, W., and Park, H. Maximum allowable delay bounds of networked control systems. *Contr. Engin. Pract.*, 2003. 11(11):1301–1313. doi:10.1016/S0967-0661(02)00238-1.

Lam, J., Gao, H., and Wang, C. H_∞ model reduction of linear systems with distributed delay. *IEE Proc. Control Theory Appl.*, 2005. 152(6):662–674. doi:10.1049/ip-cta:20045216.

Liberzon, D. Hybrid feedback stabilization of systems with quantized signals. *Automatica*, 2003. 39(9):1543–1554. doi:10.1016/S0005-1098(03)00151-1.

Matveev, A. and Savkin, A. Optimal control of networked systems via asynchronous communication channels with irregular delays. In *Proc. 40th Conf. Decision Contr.* Orlando, Florida, USA, 2001 pages 2327–2332.

Montestruque, L. and Antsaklis, P. On the model-based control of networked systems. *Automatica*, 2003. 39(10):1837–1843. doi:10.1016/S0005-1098(03)00186-9.

Montestruque, L. and Antsaklis, P. Stability of model-based networked control systems with time-varying transmission times. *IEEE Trans. Autom. Contr.*, 2004. 49(9):1562–1572. doi:10.1109/TAC.2004.834107.

Mou, S., Gao, H., Qiang, W., and Chen, K. New delay-dependent exponential stability for neural networks with time delay. *IEEE Trans. Syst. Man Cybernetics, Part B*, 2008. 38(2):571–576. doi:10.1109/TSMCB.2007.913124.

Mu, S., Chu, T., and Wang, L. Impulsive control of networked systems. *Dynam. Contin., Discr. Impuls. Syst. Proc.: Impuls. Dynam. Syst. Appl.*, 2004. 2:67–75.

Nilsson, J., Berhardsson, B., and Wittenmark, B. Stochastic analysis and control of real-time systems with random time delays. *Automatica*, 1998. 34(1):57–64. doi:10.1016/S0005-1098(97)00170-2.

Park, P. A delay-dependent stability criterion for systems with uncertain time-invariant delays. *IEEE Trans. Autom. Contr.*, 1999. 44(4):876–877. doi:10.1109/9.754838.

Tian, E., Yue, D., and Peng, C. Quantized output feedback control for networked control systems. *Information Science*, 2008. 178(12):2734–2749. doi:10.1016/j.ins.2008.01.019.

Tipsuwan, Y. and Chow, M. Control methodologies in networked control systems. *Contr. Engin. Pract.*, 2003. 11(10):1099–1111. doi:10.1016/S0967-0661(03)00036-4.

Wong, W. and Brockett, R. Systems with finite communication bandwidth constraints -II: stabilization with limited information feedback. *IEEE Trans. Autom. Contr.*, 1999. 44(5):1049–1053. doi:10.1109/9.763226.

Yu, M., L., L. W., Chu, T., and Hao, F. An LMI approach to networked control systems with data packet dropout and transmission delays. *Int. J. Hybr. Syst.*, 2003. 3:291–303.

Yue, D. and Han, Q. Network-based robust H_∞ filtering for uncertain linear systems. *IEEE Trans. Signal Process.*, 2006. 54(11):4293–4301. doi:10.1109/TSP.2006.880038.

Yue, D., Han, Q., and Lam, J. Network-based robust H_∞ control of systems with uncertainty. *Automatica*, 2005. 41(6):999–1007. doi:10.1016/j.automatica.2004.12.011.

Yue, D., Han, Q., and Peng, C. State feedback controller design of networked control systems. *IEEE Trans Circuits and Systems - II*, 2004. 51(11):640–644. doi:10.1109/TCSII.2004.836043.

Yue, D., Peng, C., and Tang, G. Guaranteed cost control of linear systems over networks with state and input quantisations. *IEE Proc. Control Theory Appl.*, 2006. 153(6):658–664. doi:10.1049/ip-cta:20050294.

Z. Wang, D. H., F. Yang and Liu, X. Robust H_∞ control for networked systems with random packet losses. *IEEE Trans. Syst. Man Cybernetics, Part B*, 2007. 37(4):916–924. doi:10.1109/TSMCB.2007.896412.

Zhang, W., Branicky, M., and Phillips, S. Stability of networked control systems. *IEEE Contr. Syst. Magaz.*, 2001. 21(1):84–99. doi:10.1109/37.898794.

Zhivoglyadov, P. and Middleton, R. Networked control design for linear systems. *Automatica*, 2003. 39(4):743–750. doi:10.1016/S0005-1098(02)00306-0.